

# The Highly Flattened Dark Matter Halo Of NGC 4244

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## ABSTRACT

In a previous paper (Olling 1995, AJ, 110, 591) a method was developed to determine the shapes of dark matter halos of spiral galaxies from an accurate determination of the rotation curve, the flaring of the gas layer and the velocity dispersion in the H I. Here this method is applied to the almost edge-on Scd galaxy NGC 4244 for which the necessary parameters are determined in the accompanying paper (AJ, Aug. 1996).

The observed flaring of the H I beyond the optical disk puts significant constraints on the shape of the dark matter halo, which are almost independent of the stellar mass-to-light ratio. NGC 4244's dark matter halo is found to be highly flattened with a shortest-to-longest axis ratio of  $0.2^{+0.3}_{-0.1}$ . If the dark matter is disk-like, the data presented in this paper imply that the vertical velocity dispersion of the dark matter must be 10% - 30% larger than the measured tangential dispersion in the H I. Alternatively, the measured flaring curve is consistent with a round halo if the gaseous velocity dispersion ellipsoid is anisotropic. In that case the vertical dispersion of the gas is 50 - 70% of the measured tangential velocity dispersion.

## 1. INTRODUCTION

Although rotation curves of spiral galaxies have been used as evidence for the presence of dark matter (DM), little is known about the nature, extent and actual distribution of the DM halos (e.g., van Albada *et al.* 1985; Lake & Feinswoog 1989). As measurements of the equatorial rotation curve probe the potential in only one direction, they provide no information about the shape of the DM halos.

In a previous paper, Paper I (Olling 1995), a method was developed to determine the shape of the dark matter halo from the gaseous velocity dispersion and the radial variation of the thickness of gas layers (flaring). This is accomplished by comparing the measured flaring with that expected from a self-gravitating gaseous disk in the axisymmetric potential due to the stellar disk and (flattened) DM halo. The method of calculation which produces the most reliable model flaring curve is the *global* approach, in which the potential is calculated from the *total* mass distribution of the galaxy (Paper I). In this paper the method is applied to the galaxy NGC 4244 for which the basic parameters were determined in the accompanying paper (Olling 1996, hereafter Paper II).

In the past, several methods have been used to determine the shapes of dark matter halos. Hofner & Sparke (1994) analyzed the warping behavior of H I disks and concluded that only one (NGC 2903) of the five systems studied requires a DM halo as flattened as E4<sup>1</sup>. On the other hand, in studies of polar ring galaxies (Sackett & Sparke 1990; Sackett *et al.* 1994; Sackett & Pogge 1995) substantially flattened DM halos are found (E6-E7 for NGC 4650A, E5 for A0136-0801). The shape of the dark halo of the Milky Way has been estimated (E0 - E7) from the kinematics of extreme Population II stars (Binney & Ostriker 1987b; Sommer-Larsen & Zhen 1990; van der Marel 1991; Amendt & Cuddeford 1994). Steiman-Cameron *et al.* (1992) used the dynamics of the precessing dusty disk of the S0 galaxy NGC 4753 to infer the shape of its DM halo (E1).

<sup>1</sup>A spheroidal system with shortest-to-longest axis ratio  $c/a$  ( $q_\rho$ ) of the density contours has a shape En, with  $n$  such that  $q_\rho = 1 - n/10$

Van der Kruit (1981) pioneered the use of flaring measurements to determine the mass of stellar disks, and found that the scale length of the total matter (luminous plus dark) was similar to the scale length of the light distribution and concluded that the mass-to-light ratio does not vary significantly with radius. Since mass models where all the mass is concentrated in the disk require large radial changes in the mass-to-light ratio ( $M/\mathcal{L}$ ) to reproduce the observed flat rotation curves, the DM halo is not as highly flattened as the stellar disk. A similar conclusion was reached for NGC 4565, NGC 891 (Rupen 1991), and the Milky Way (Knapp 1987; Merrifield 1992; Malhotra 1994, 1995). As we can measure the light distribution of the stellar disk only and have no prior knowledge of the mass-to-light ratio of stellar disks, the relative contributions of luminous and dark matter to a given rotation curve are not known. Models of galaxy formation which include simple estimates of the star formation history (e.g., Larson & Tinsley 1978) predict  $M/\mathcal{L}$ 's which are in the range of values derived from observed rotation curves (e.g., Rubin *et al.* 1985; Athanassoula *et al.* 1987; Bottema 1988; Broeils 1992, Chap. 12), but different star formation histories can easily result in mass-to-light ratios which differ by a factor of a few (e.g., Tinsley 1981; Worthey 1994; de Jong 1995, Chap. 4). An upper limit to the mass-to-light ratio, and hence a lower limit to the amount of DM, can be obtained by assuming that the peak of the observed rotation curve is due to the stellar disk only, the so called maximum-disk hypothesis (van Albada & Sancisi 1986). Other  $M/\mathcal{L}$  estimators have been used (van der Kruit 1981; Efstathiou *et al.* 1982; Athanassoula *et al.* 1987; Martinet 1988; Bottema 1993), typically yielding a stellar disk 50% - 100% as massive as the "maximum-disk". Employing the maximum-disk model, Broeils (1992, 1995) finds that the ratio of dark to luminous matter lies between 0.5 and 10, where the larger values are found in low mass systems. Unfortunately, the flaring data presented in this paper cannot be used to constrain the mass of the stellar disk of NGC 4244 (Sec. 3.3.) beyond the limits imposed by the shape of the rotation curve.

In a study similar to the present one, Rupen (1991)

analyzed high resolution H I observations of two edge-on galaxies, NGC 4565 and NGC 891. For these galaxies he finds that the flaring data imply stellar disks with masses two-third and one-half as massive as a “maximum-disk”, consistent with van der Kruit’s (1981, 1988) results for NGC 891. In addition, Ruken found that non-thermal pressure gradients ( $\frac{dP_{nt}}{dz}$ ) need probably not be included in the equation of hydrostatic equilibrium (1). The fact that NGC 891 and NGC 4565 have similar luminous mass distributions, similar gas layer widths and similar thermal pressure gradients ( $\frac{dP_{th}}{dz}$ ) but are estimated to have very different non-thermal pressure gradients ( $\left.\frac{dP_{nt}}{dz}\right|_{NGC4565} \approx \frac{1}{2}; \left.\frac{dP_{nt}}{dz}\right|_{NGC891} \approx 4$ ) argues against the importance of non-thermal pressure gradients in balancing the vertical force.

Below I summarize the relevant properties of NGC 4244 as determined in Paper II and references therein. The current starformation rate in NGC 4244 is low. Apart from a region between 6.5 and 8 kpc to the south-west, the radial light profile is well represented by an exponential distribution: extinction is probably not very important. The properties of the neutral hydrogen layer were derived from sensitive high resolution VLA<sup>2</sup> observations. The total H I distribution is fairly regular and shows a gentle warp. Several strong H I surface density variations are seen in the inner disk. Beyond the optical disk the surface density on both sides of the galaxy is similar, validating the use of the average surface density profile in the model calculations. The same holds for the gaseous velocity dispersion, and the inferred inclination angle. Superimposed upon a gradual increase of the gas layer width, the derived thickness of the gas layer shows a characteristic increase/decrease on the inside/outside of spiral arms. These features are probably not real but are likely an artifact (induced by spiral arm streaming motions) of the thickness-and-inclination derivation technique (Paper II). NGC 4244 is an intermediate mass system with a rotation speed of about 100 km s<sup>-1</sup>. The large inclination ( $\sim 84^\circ$  5)

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<sup>2</sup>The VLA of the National Radio Astronomy Observatory is a facility of the National Science Foundation operated under cooperative agreement by Associated Universities, Inc.

of NGC 4244 in combination with the symmetric surface density distribution, allowed for the accurate determination of the rotation curve. NGC 4244 is one of the few galaxies to show, on both sides, the characteristic drop in rotation velocity beyond the optical disk. This decline in rotation speed is more typical of luminous fast rotators (Casertano & van Gorkom 1991). The slow rise in the inner part and the decline of the rotation curve beyond the optical disk limit the mass-to-light ratio of NGC 4244 to 50% - 100% of the maximum-disk value. Due to the limited extend of the measured rotation curve the dark halo’s core radius ( $R_c$ ) and central density ( $\rho_0$ ) could be determined to within a factor of 50 and 900 only, respectively. This large uncertainty in the dark-halo’s density distribution does not preclude the determination of the halo’s shape (Paper I).

In Sec. 2. I describe how, in conjunction with self-consistent mass models, the observed flaring can be used to constrain the shape of the dark matter halo. The shape of the halo is then determined in Sec. 2.1.. Some caveats related to various step along the way are discussed in Sec. 3.. The implications of the highly flattened halo for the amount and nature of dark matter are discussed in Sec. 4..

## 2. THE MASS MODEL

The dependence of the thickness of the gas layer upon the shape of the dark matter halo is described in Paper I. Here I will only present four of the most salient features. That the thickness of a gas layer depends on the axial ratio ( $q_\rho = c/a$ ) of a dark halo can be easily understood. For an axisymmetric ellipsoidal dark matter distribution with fixed  $R_c$  and  $\rho_0$  [like Eq. (2)], the mass enclosed by an ellipsoid of axial ratio  $q_\rho$ , and hence the rotation speed, decreases with decreasing  $q_\rho$ . Thus, for a given observed rotation curve, the densities of flattened halos have to be larger than for a round halo . The vertical force, proportional to the mass enclosed (Gauss’ theorem), increases as well so that flattened halos have thinner gas layers. The second important result obtained in Paper I is that the mass-to-light ratio (or equivalently, the mass) of the stellar disk has virtually no influence on the thickness of the gas layer beyond the stellar

disk. Given the fact that the dark matter density distribution is poorly known (van Albada *et al.* 1985; Lake & Feinswog 1989; Paper I; Paper II) this may seem surprising at first. However, close to the plane the vertical force is roughly proportional to the radial force<sup>3</sup> which is, by construction, the same for all disk-halo combinations that reproduce the observed rotation curve: beyond the optical disk the dependence on halo flattening dominates. Thirdly, the self-gravity of the gas can contribute significantly to the vertical force beyond 2 - 3 optical scale lengths. And finally, the model gas layer widths are best calculated using the global approach, in which the vertical force is determined from the global mass distribution of the galaxy.

Comparing the thickness measurements beyond the optical disk with model flaring curves, calculated for a series of mass models with varying halo flattening, yields the flattening of the DM halo. The basic assumptions are that the vertical force  $K_z(z)$  is balanced by gradients in the thermal pressure ( $P_{\text{th}}$ ) and that the gaseous velocity dispersion ellipsoid is round and independent of height above the plane, so that the vertical Jeans equation [e.g., Paper I, Eq. (1)] can be approximated by the equation of hydrostatic equilibrium:

$$\frac{dP_{\text{th}}}{dz} = \rho_{\text{gas}} K_z . \quad (1)$$

Input to these mass models are the gaseous velocity dispersion, the observed rotation curve, and the density distributions of the dark and luminous matter. Below I describe how to arrive at these density distributions.

Optical studies of edge-on spiral galaxies indicate that the vertical scale height of the stellar disk is constant as a function of galactocentric distance (van der Kruit & Searle 1981a,b 1982a,b; Shaw & Gilmore 1990; Barteldrees & Dettmar 1994), although there are theoretical and observational indications that the shape of the vertical distribution may change with radius (Burkert *et al.* 1992 and van Dokkum

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<sup>3</sup> $K_z = z/\sqrt{z^2 + R^2}$   $F_{\text{tot}} \approx z/R F_R \propto \frac{z}{R} V_{\text{obs}}^2$

*et al.* 1994, respectively). Here a constant vertical scale height is assumed. For the vertical distribution I use an exponential as well as a  $\text{sech}^2(z/z_0)$  distribution. The effects of a different choice for the vertical stellar distribution are discussed in Sec. 3.3.1.. To avoid extinction features (Paper II), a fit to the stellar radial surface brightness distribution is used:  $L(R) = L(0) \exp(-R/h_R)$  for  $R \leq 10$  kpc, followed by a linear decline till 11 kpc and  $L(R) = 0$  beyond 11 kpc. A constant mass-to-light ratio ( $M/\mathcal{L}_B$ ) is assumed. The stellar density distribution is thus given by the product of the mass-to-light ratio and the adopted radial and vertical distributions.

The gaseous surface density used is the sum of the H I surface density distribution, a crude estimate of the H<sub>2</sub> surface density distribution, and 25% He by mass (Paper II). The *volume* density is found from  $\Sigma_{\text{gas}}(R)$ ,  $\sigma(R)_{\text{gas}}$  and the total potential, via the equation of hydrostatic equilibrium. Details of this procedure can be found in Paper I. A Gaussian fit to this density distribution yields the model gas layer widths (but note that deviations from the Gaussian distribution occur inside the optical disk, see footnote 4 in Paper I). In Sec. 2.1. these model widths will be compared to the measured flaring curve.

The dark halo volume density is parameterized as a flattened isothermal sphere with core radius  $R_c(q_\rho)$ , central density  $\rho_0(q_\rho)$ , and flattening  $q_\rho (= c/a)$  (e.g., Sackett & Sparke 1990; Sackett *et al.* 1994; Paper I):

$$\rho(R, z; q_\rho) = \frac{\rho_0(q_\rho) R_c^2(q_\rho)}{R_c^2(q_\rho) + R^2 + (z/q_\rho)^2} . \quad (2)$$

The dependency of  $R_c$  and  $\rho_0$  upon  $q_\rho$  is such that the rotation curve of any flattened density distribution described by Eq. (2) is practically indistinguishable from its round equivalent (Paper I). In the flat rotation curve regime, the thickness of the gas layer in a density distribution like Eq. (2) is given by:

$$FWHM_z(R) \approx 2.35 \left( \frac{\sigma_{\text{gas}}}{V_{\text{rot,max}}} \right) \times \sqrt{\frac{2.4 q}{1.4 + q}} \sqrt{R^2 + R_c^2} , \quad (3)$$

with  $V_{\text{rot},\text{max}}$  the asymptotic rotation speed of the DM halo [Paper I, Eq. (D31)].

As the mass models described above are fully specified by two independent free parameters, the stellar mass-to-light ratio and the flattening of the DM halo, the flaring behavior can be investigated as a function of these parameters. For every choice of the stellar  $M/\mathcal{L}$ , the DM core radius and central density are uniquely determined by two points on the observed rotation curve (e.g., at  $2.3h_R$  and  $8h_R$ , see Paper I, Appendix B). However, the predicted flaring beyond the optical disk is essentially independent of the choice of  $M/\mathcal{L}$  (Paper I; Fig. 1). The shape of the DM halo is not a parameter in such rotation curve decompositions, although different halo flattenings require somewhat different values of  $R_c$  and  $q_\rho$  [Paper I, Eqs. (B8) and (B9)]. In Paper II it was found that the DM-halo parameters are not well constrained: possible core radii range from 0.3 to 12 kpc and central densities between 1.3 and  $0.001 M_\odot/\text{pc}^3$  are allowed; small  $M/\mathcal{L}$ 's correspond to small core radii and large halo densities. Again, notwithstanding this enormous allowed range in halo parameters, the thickness of the gas layer in such disk-halo models (with the same flattening) is very similar.

The Dark Matter halo model [Eqn. (2)] is not unique (e.g., van Albada *et al.* 1985; Lake & Feinswog 1989; Navarro, Frenk & White 1996). However, all DM density distributions with similar rotation curves share the feature that flatter distributions have larger vertical forces and hence thinner gas layers. Thus, the flattening determined in this paper is an indicator of the true flattening of the true DM density distribution. In the discussion (Sec. 4.) I investigate the extreme possibility that the dark matter is disk-like.

## 2.1. The Shape of the Dark Halo

Now we can compare the calculated gas scale heights for several mass models with various  $M/\mathcal{L}$  and  $q_\rho$  with the observed widths (Fig. 1). The truncation of the stellar disk, the gaseous self-gravity and the measured gaseous velocity dispersions have been included in the model calculations. Inspecting this figure we notice general correspondence between the observations and the model flaring curves, except for

the 7 kpc hump, where the thickness of the gas layer increases by a factor 2.7 within 1 kpc (on both sides of the galaxy). Similar features are seen in the Milky Way (Knapp 1987) and NGC 891 (Rupen 1991) but are absent in NGC 4565 (Rupen 1991). The steep thickness gradients in NGC 891 and NGC 4244 seem to occur just inside and just outside H I surface density enhancements. Such sudden changes in apparent thickness can arise from streaming motions induced by spiral structure (Paper II, Appendix C.).

For the best-fit halo flattening, the maximum-disk and the minimum-disk model curves seem to represent the flaring data equally well (upper and lower panel, respectively). This is corroborated by the fact that the model with the largest reduced  $\chi^2$  lies within the  $1-\sigma$  bound (i.e.,  $\chi^2 \leq \chi^2_{\text{min}} + 1$ ) of the model with the smallest reduced  $\chi^2_{\text{min}}$ . The north-eastern, south-western, and the average flaring curves all yield the same estimate for the mass-to-light ratio. Thus, for NGC 4244, the measured flaring does not constrain the mass-to-light ratio of the stellar disk beyond the limits set by the shape of the rotation curve.

On the other hand, the flattening of the halo is much better constrained. For all stellar disk masses, the minimum  $\chi^2$  is found at the same halo flattening, with the same formal uncertainty. The uncertainty in the inclination determination in the outer parts of the galaxy (Paper II) has only a small effect on the inferred DM-halo shape: for the varying-inclination case I find  $q_\rho = 0.2 \pm 0.1$ , the fixed-inclination case yields  $q_\rho = 0.3^{+0.2}_{-0.1}$ . There are several reasons why I believe that the small decrease in inclination beyond the optical disk is probably real: 1) it is seen on both sides of the galaxy, 2) it is seen (with lower S/N) when using the low velocity channels (8-14) only, as well as in the “edge” channels (15-20) only. Nevertheless, I include the fixed-inclination case in the final solution:  $q_\rho = 0.2^{+0.3}_{-0.1}$ .

## 3. CAVEATS

We have seen that the dark halo of NGC 4244 is significantly flattened. Below I discuss the main uncertainties in this result.

### 3.1. The Gaseous Velocity Dispersion

As the calculated thickness of the gas layer and the inferred halo flattening depend, respectively, linearly and inversely quadratically on the velocity dispersion of the gas<sup>4</sup>, the velocity dispersion of the gas is an important parameter in the analysis presented above. This strong dependence is illustrated in Fig. 2, where I compare model gas layer widths calculated for the lower and upper bounds to the gaseous velocity dispersion found in other, more face-on, systems (van der Kruit & Shostak 1982; Dickey *et al.* 1990; see Kamphuis 1993, Chap. 12, for a review). It is striking that the shape of the halo cannot be determined if no velocity dispersion *measurements* are available: measuring the gaseous velocity dispersion is instrumental in the determination of the shape of the dark matter halo. From Fig. 3 we can see that forcing the flaring measurements to be consistent with a round DM-halo model requires a radial decline of the velocity dispersion which is inconsistent with the dispersion measurements. Given that line-of-sight smearing effects are unimportant and that the data from which the tangential velocity dispersion was determined has a high signal to noise ratio ( $S/N \geq 10$  for  $R \leq 14$  kpc; Paper II), it is highly unlikely that the measured velocity dispersion curve is consistent with the velocity dispersion needed to infer a round halo.

The flaring data can be consistent with a round halo if the vertical dispersion ( $\sigma_{zz}$ ) does not equal the measured tangential dispersion ( $\sigma_{\theta\theta}$ ). The measured tangential dispersion curve can thus be consistent with a round DM halo if  $(\sigma_{zz}/\sigma_{\theta\theta})$  decreases radially (down to 0.55 at the last measured point). However, it is unclear whether such a large anisotropy in the velocity dispersion of the H I beyond the optical disk is consistent with the observations. For the molecular component in the solar neighborhood this ratio is not well established:  $\sigma_{zz} = 5.7 \pm 1.2$  for high latitude CO clouds (Blitz *et al.* 1984) while determinations of  $\sigma_{\theta\theta}$  from tangent point measurements vary between  $3 \pm 0.7$  (Clemens 1985) and  $7.8 \pm 4$  (Malhotra 1994).

Assuming that an anisotropy in the gaseous ve-

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<sup>4</sup>From Eq. (3) it follows that  $FWHM_z^2 \propto \sigma_{\text{gas}}^2 q_\rho$ .

locity dispersion will be reflected in the motions of young stars formed from this gas, proper motion studies of OB associations may be informative. On very small scales ( $\sim 0.5$  pc) there is some evidence that the velocity dispersion ellipsoid in the Orion Nebula Cluster is anisotropic ( $\sigma_b/\sigma_l = 2.2/2.5 = 0.9 \pm 0.1$ ), while it is uncertain how much this value is affected by the expansion of the cluster (Jones & Walker 1988). On larger scales ( $\sim 20$  pc), the anisotropy of a proper motion selected sample of stars is similar ( $2.9/3.2 = 0.9 \pm 0.3$ ; McNamara *et al.* 1989), while a large proper motion and distance selected sample (Warren & Hesser 1977) covering a large volume ( $175^3$  pc<sup>3</sup>) centered on Orion OB1 shows an increase of dispersion values but no evidence for a velocity dispersion anisotropy ( $17.3/16.7 = 1.04 \pm 0.2$ ). Mihalas & Binney (1981, p. 423) report a small anisotropy in the dispersion for O-B5 stars ( $\sigma_{\theta\theta}/\sigma_{zz} = 11/9 = 1.2$ ) with an unknown error. In summary, the velocity dispersion ellipsoid of young OB stars, and presumably of the interstellar medium, is not significantly anisotropic in the solar neighborhood.

Possibly the best way to determine the velocity dispersion ellipsoid, is by comparing the velocity dispersions in a sample of face-on and edge-on galaxies. We (van Gorkom, Rupen & Olling) are in the process of doing so.

### 3.2. The Determination of the Inclination

Because the inferred halo flattening is proportional to the square of the true width of the gas layer [Eq. (3)], the thickness of the gas layer has to be measured reliably (Paper II). The difference in the inferred  $q_\rho$  between the thickness for the fixed-inclination case ( $q_\rho = 0.3^{+0.2}_{-0.1}$ ) and the free-inclination case ( $q_\rho = 0.2 \pm 0.1$ ) exceeds the  $q_\rho$ -error resulting from the errors in the *measured* gaseous velocity dispersions by a factor of two.

### 3.3. The Stellar Mass Distribution

#### 3.3.1. The vertical distribution

Since the shape of the DM halo can be determined irrespective of the actual mass of the stellar disk, its

vertical distribution is unimportant as well.

### 3.3.2. Does light trace mass ?

In Sec. 2. it was assumed that the stellar mass-to-light ratio is constant throughout the galaxy. As argued above, a vertically varying mass-to-light ratio does not affect the inferred halo flattening.

A radial gradient in the stellar  $M/\mathcal{L}$  would change the radial distribution of stellar mass. Consider the simple case where the true radial scale length differs from the value used for  $h_R$ ; this difference would expand the horizontal scale of the *predicted* flaring curves by a factor  $h_{R,\text{true}}/h_{R,\text{false}}$ , while the measured flaring remains unchanged. Thus, the usage of too long a radial scale length leads to an overestimate of the halo flattening (too round a halo). In many early type spirals the scale length as determined from near-infrared photometry is significantly shorter than the scale length at visual wavelengths (Terndrup *et al.* 1994; Peletier *et al.* 1994). Extinction can not be the sole reason for this radial blueing since such color gradients are found in face-on systems as well (de Jong & van der Kruit 1994; de Jong 1995). In fact, it may be that a radially varying star formation history (mean stellar age) in combination with a radial metallicity gradient cause the observed color gradients (Josey & Arimoto 1992; de Jong 1995, Chapter 4). However, such color gradients are small<sup>5</sup> for late type, low luminosity systems like NGC 4244. Since extinction is not very important in NGC 4244 and the current star formation is low, the measured blue light probably traces mass, so that the inferred DM-halo flattening is probably unaffected.

## 3.4. Distance Dependence

By combining the equations (D30) and (B16) from Paper I, it follows that the model thickness, inside the stellar disk, is proportional to  $\sqrt{h_R z_e}$ , equivalent to a linear distance dependence. Likewise, the the thick-

<sup>5</sup>There are twelve systems in de Jong's (1995) sample which are comparable to NGC 4244: in scale length, total luminosity and total H I mass. For those systems I find  $h_{\text{K-band}}/h_{\text{B-band}} = 1.02 \pm 0.06$ , corresponding to small color gradients.

ness of the gas layer beyond the optical disk is linearly proportional to galactocentric radius [Eq. (3)], thus depending linearly on distance. As the linear scale of the thickness measurements depend linearly on distance as well, the flaring gas layer method yields distance-independent values for the mass-to-light ratio of the stellar disk and the shape of the DM halo.

## 3.5. The Influence of the Extragalactic Radiation Field

The H I surface density profile of M33 (Corbelli *et al.* 1989) and the major axis profile of NGC 3198 (van Gorkom 1993) show sudden change in slope at column densities below a few times  $10^{19} \text{ cm}^{-2}$ . Maloney (1993) models the NGC 3198 case extensively and argues that these slope changes are evidence for the extragalactic radiation field (EgRF) ionizing the neutral hydrogen layer. As a result, the degree of ionization will increase with height above the plane as well, so that width of the *neutral* gas layer ( $\text{FWHM}_{\text{HI}}$ ) will be smaller than the thickness of the column of total hydrogen ( $\text{FWHM}_{\text{tot}}$ ), at low column densities. Since the low column density regime generally coincides with the region where the thickness of the H I layer is sensitive to the shape of the DM halo, I investigate the effects of the EgRF on the width of the gas layer. Inspection of Maloney's Figs. 4b and 7, and comparing the densities of the NGC 3198 model with the measured densities in NGC 4244, indicates that the thickness of NGC 4244's H I layer may be underestimated by  $\sim 15\%$  at 12 kpc. However, Maloney's models were calculated for an EgRF,  $\Phi_{i,4}$ , of  $10^4$  photons  $\text{cm}^{-2} \text{ s}^{-1}$  while the 2- $\sigma$  upper limit is three times larger (Vogel *et al.* 1995). Assuming that the effects of the radiation field are similar if the ratio of critical density<sup>6</sup> ( $N_c^{\text{HI}}$ ) to H I surface density is similar, I used Maloney's NGC 3198 model to determine the dependence of the ratio of observed to total thick-

<sup>6</sup>In an analytic approximation of the effects of the EgRF, Maloney (1993) finds that at total column densities below the critical density, the whole hydrogen column is ionized, while his detailed analysis indicates a rapid change of fractional ionization for column densities near  $N_c^{\text{HI}}$  [ $= 4.45 \cdot 10^{19} \sqrt{\Phi_{i,4} \text{ FWHM}_{\text{HI}}} \text{ cm}^{-2}$ , adapted from Maloney's Eqn (19)].

ness ( $r_{\text{FWHM}}$ ) on the ratio of critical to H I surface density ( $r_\Sigma$ )<sup>7</sup>. With this relation it is possible to correct the observed thickness for the effects of the extragalactic radiation field, for any observed thickness and H I column density, and assumed EgRF strength. For the radii of interest I find that the thickness of the total hydrogen is  $(3.4, 7.4, 18.4, 23.7, 35.4) \times \Phi_{i,4}$  % larger than the measured thickness of the H I layer at 10.5, 11, 11.5, 12, and 12.5 kpc, respectively. For small values of the radiation field,  $\Phi_{i,4} < 2$ , the thickness corrections are small so that inferred halo flattening is not affected. For stronger radiation fields ( $\Phi_{i,4} = 2\text{-}3$ ) we find:  $q_\rho = 0.5 \pm 0.2$ . In summary, the effects of the extragalactic radiation field on the determination of  $q_\rho$  are at most of the same order of magnitude as the uncertainties in the gaseous velocity dispersion and inclination.

### 3.6. Non-Thermal Pressures Gradients

The analysis presented in this paper assumes the vertical gravitational force is balanced by thermal pressure gradients only. It is estimated that in the solar neighborhood the pressures due to cosmic rays, magnetic fields and kinetic gas pressure are of comparable magnitude (Spitzer 1978, p. 234). The magnetic field of the Milky Way has a very long radial scale length (Kulkarni & Heiles 1988, p. 146) and could provide significant pressure, even beyond the optical disk. However, such a stratified magnetic field is unstable (Parker 1966) and may therefore not contribute to the vertical pressure balance. Non-thermal radio emission indicates that cosmic rays are closely related to sites of star formation (Bicay & Helou 1990), and do not extend much beyond such regions. With NGC 4244's radio continuum flux very low (Paper II) it is expected that cosmic ray pressure and/or magnetic field strength contributes little to the total pressure, especially beyond the optical disk. In the Introduction I summarized observational evidence against the

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<sup>7</sup>With

$r_\Sigma = N_c^{\text{HI}} / \Sigma_{\text{HI}}$ , and  $r_{\text{FWHM}} = (\text{FWHM}_{\text{HI}} / \text{FWHM}_{\text{tot}})$ , I find that the following relation holds:  $r_{\text{FWHM}} \approx 1 + 2.57(1 - \exp(-(r_\Sigma/1.35)^{2.2})) \pm 0.015$  for  $r_\Sigma \leq 3$ , while for larger  $r_\Sigma$ -values  $r_{\text{FWHM}}$  decreases again, similar to Maloney's Fig. 7.

importance of non-thermal pressures (Rupen 1991). At any rate, if non-thermal pressures are important, then an even larger DM-halo midplane density is required to produce the observed flaring: additional non-thermal pressures require an even flatter DM halo (see also Pfenniger *et al.* 1994, and references therein).

### 3.7. Combined systematic effects

Having established that the velocity dispersion ellipsoid of the H I is probably round (Sec. 3.1.), I like to stress once more the importance of determining the thickness of the gas layer and the velocity dispersion accurately. Furthermore, due to projection effects, the derived thickness of the gas layer is quite sensitive to the exact value of the inclination (for  $i=85^\circ \pm 2^\circ$ ; Paper II). I illustrate this by comparing the inferred DM halo flattenings for the case that gas layer widths are determined assuming a constant inclination of  $84.5^\circ$ , and the case where the whole spectral line cube is used to determine thickness and (varying) inclination simultaneously. If the galaxy were exactly edge-on or at an inclination  $\lesssim 80^\circ$  but larger than  $60^\circ$ , the uncertainty in the derived gas layer width would be much less severe (Paper II).

For example, in the varying (fixed) inclination case the dispersion has to be decreased by 6.2 (3.2) times the errors on the dispersion measurements<sup>8</sup> on all 9 (10) independent points between 8 and 13 kpc to be consistent with a round halo ( $4.1\sigma$  and  $1\sigma$  times smaller for a  $q_\rho=0.5$  halo). Similarly, in order to infer a round halo, the thickness of the gas layer (as presented in Fig. 1) has to change by  $+3.4\sigma_{\text{FWHM}}$  ( $+2.5\sigma_{\text{FWHM}}$ ) for the varying (fixed) inclination case ( $+2.1\sigma_{\text{FWHM}}$  and  $+1\sigma_{\text{FWHM}}$  for a  $q_\rho=0.5$  halo). Combining these systematic effects, I find that a round halo requires a  $2.2\sigma$  ( $1.7\sigma$ ) change for the varying (fixed) inclination case and a  $1.5\sigma$  ( $0.5\sigma$ ) change for a  $q_\rho=0.5$  halo. We see that it is highly unlikely that the systematic offsets from the measured values of  $\sigma_{\text{gas}}$  and  $\text{FWHM}_z$  required for consistency with a

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<sup>8</sup>Alternatively, a  $1\sigma$  systematic dispersion offset can be thought of as a velocity dispersion anisotropy of  $\sim 8\%$ .

round halo, or even for a halo of intermediate flattening, are chance occurrences.

Taking the uncertainties due to the extragalactic radiation field (Sec. 3.5.) into account, the required systematic offsets in dispersion and thickness to infer a rounder DM halo become smaller. A moderately flattened halo with  $q_\rho = 0.5$  is consistent with the fixed inclination case for all but the smallest  $\Phi_{i,4}$ -values, while a round halo would be inferred if measured dispersion values are systematically decreased by  $1/2$ times the dispersion errors. However, when using the preferred varying inclination case, a systematic change by  $0.9\sigma$  ( $1.5\sigma$ ) of the measured dispersions and gas layer widths is required in order to infer a  $q_\rho=0.5$  (round) DM halo.

To summarize, assuming only random errors in inclination, velocity dispersion, gas layer width and extragalactic radiation field leads to the conclusion that the DM halo of NGC 4244 is highly flattened:  $q_\rho = 0.2 \pm 0.1$ . An extragalactic radiation field equal to the current  $2-\sigma$  upper limit increases this value to  $q_\rho = 0.3 \pm 0.2$ . A velocity dispersion anisotropy of 25% (50%) would be required to increase  $q_\rho$  to 0.5 (1.0), essentially independent of the value of the extragalactic radiation field.

#### 4. DISCUSSION

The gas layer width increases exponentially with radius if the potential is dominated by the stellar disk (e.g., van der Kruit 1988; Paper I). For this reason, most authors fit exponential functions to the observed flaring curves and use simple models for the potential of the galaxy to derive that the stellar mass-to-light ratio is roughly constant with values between 50 and 100% of the maximum disk value (van der Kruit 1981; Rupen 1991; Paper II). A more detailed model of the potential, like the one described in Paper I, could be used to re-analyse these data sets. However, in order to improve upon the current  $M/\mathcal{L}$  estimates, both the vertical stellar distribution, and the gaseous velocity dispersion need to be accurately determined. For all the extra-galactic systems, the flaring measurements do not extend beyond the optical disks, so that no information regarding the shape of the dark halo could be obtained. With the edge of the stellar disk of

the Milky Way at  $\sim 14$  kpc (Robin *et al.* 1992) some of the H I surveys extend into the halo of the Galaxy. The interpretation of this data however would be difficult because of the uncertainties in the rotation curve (e.g. Merrifield 1992) and the gaseous velocity dispersion.

Dissipationless galaxy formation simulations tend to produce triaxial DM halos (e.g., Frenk *et al.* 1988; Warren *et al.* 1992), with an intrinsic flattening distribution peaked at  $q_\rho = c/a = 0.5 (\pm 0.15)$ . Adding 10% baryonic matter (gas) and including gasdynamics to these simulations produces more oblate halos while  $q_\rho$  remains unchanged (Katz & Gunn 1991; Udry & Martinet 1994; Dubinski 1994). Observationally, all but the flattest shapes have been reported (for a summary, see Sec. 1.). Significantly flattened halos suggest that the process of galaxy formation is more dissipational, which might occur if a larger fraction of the initial density perturbation is baryonic. Since the flaring can be determined at lower inclinations as well, possibly at inclinations as low as  $60^\circ$ , the shape of dark matter halos surrounding many galaxies of different Hubble types can be determined. It will then be possible to gauge the significance of the highly flattened dark halo of NGC 4244, and place more stringent constraints on the process of galaxy formation.

Rich clusters contain  $\sim (10 \pm 5)h^{-1.5}$  % (by mass) hot X-ray emitting gas (Briel *et al.* 1992; Mushotzky *et al.* 1995, and references therein), with  $h$  the normalized Hubble constant ( $h = H_0/(100 \text{ km s}^{-1}\text{Mpc}^{-1})$ ). It might thus be useful to explore the process of galaxy formation over a larger range of baryon fractions. With a total mass-to-light ratio of  $(200 - 300)h$  (Bahcall *et al.* 1995) for these rich clusters, the mass-to-light ratio of cluster baryons ( $M/\mathcal{L}_{\text{Bar,cl}}$ ) equals  $(32 - 48)h^{-0.5}$ . Standard Big Bang nucleosynthesis (BBN) models (e.g., Walker *et al.* 1991) limits  $M/\mathcal{L}_{\text{Bar,BBN}}$  to the range  $(11 - 35)h^{-1}$ . From these limits it follows that for  $h \geq 0.5$  essentially all baryons in the universe are located in rich clusters. In the region where the cluster and BBN estimates overlap ( $h \geq 0.2$ ),  $M/\mathcal{L}_{\text{Bar}} \sim (35 \pm 6)h^{-1}$ . The dynamical mass-to-light ratios of individual galaxies (Broeils 1992, 1995) range from 4 to 100  $h$ , with 80% of the

systems between 6 and 20  $h$ . It is thus possible that the dark halos of individual galaxies consist mainly of non-baryonic dark matter, but a 100% baryonic dark halo is also possible (Gott *et al.* 1974; Briel *et al.* 1992; Rubin 1993; Bahcall 1995; Sackett 1995). It is not clear however where, and in what form, these baryons reside in the galactic halos since all plausible forms of baryonic dark matter seem to be excluded (Hegyi & Olive 1986).

Pfenniger *et al.* (1994) reviewed cold, rotationally supported, molecular hydrogen as a dark matter candidate. In their model, small high density molecular “clumpuscules” form the building blocks of a highly clumped interstellar stellar medium (ISM) in which star formation is suppressed as a result of frequent collisions between the clumpuscules. Their model is best developed in the region beyond the optical disk. Inside the optical disk the state of the ISM is much more complex as a result of increased radiation field and metallicity. With the surface brightness of the cosmic background radiation equal to  $\sim 8.5 L_{\odot} \text{pc}^{-2}$ , the transition between the two regimes is expected to occur around  $R \sim D_{25}/2$ . The fact that in many galaxies the shape of the rotation curve due to the gas is similar to the observed rotation curve (Bosma 1981; Carignan *et al.* 1990; Carignan & Puche 1990; Broeils 1992, Chap. 10; Paper II) could then be explained if 3 - 10% of the gaseous surface density is in atomic form. Such might be expected in the context of clumpuscules hypothesis, or other cold-gas dark matter models (e.g., Gerhard & Silk 1995).

Since the self-gravity of the gas layer beyond the optical disk strongly affects the flaring (cf. Paper I, Sec. 3.3 and Fig. 8), I investigate whether the clumpuscules hypothesis is consistent with the flaring data presented in this paper. Penny Sackett kindly provided the disk-like surface density distribution inferred from NGC 4244’s rotation curve using a Keplerian as well as a flat extrapolation beyond the last measured point ( $\Sigma_{\text{tot,Kep}}$  and  $\Sigma_{\text{tot,Flt}}$ ; see also Sackett 1995). The inferred DM surface density is affected by the way the rotation curve is extrapolated:  $\Sigma_{\text{DM,Kep}} \approx 86 \exp(-R/5.2)$ , and  $\Sigma_{\text{DM,Flt}} \approx 55 \exp(-R/11.4) M_{\odot} \text{pc}^{-2}$ , but is almost independent of the stellar mass-to-light ratio. The dark-to-

H I surface density rises in exponential fashion (with scale length  $\sim 1.7 \pm 0.2$  kpc) from  $\sim 10$  at the edge of the optical disk to  $\sim 100$  at the edge of the H I disk. This increase in dark-to-H I ratio is mostly due to the declining H I surface densities below column densities of  $\sim 2 \times 10^{20} \text{ cm}^{-2}$ . Carignan & Puche (1990) find a similar effect for NGC 7793 where  $\Sigma_{\text{DM}}/\Sigma_{\text{H I}}$  increases exponentially for H I column densities below  $5 \times 10^{20} \text{ cm}^{-2}$ . This contrasts Bosma’s (1981) finding that, for column densities above  $\sim 10^{20} \text{ cm}^{-2}$ , the dark-to-H I surface density ratio is approximately constant<sup>9</sup>.

The thickness the clumpuscule disk can be calculated<sup>10</sup> when a vertical velocity dispersion is assumed. I find that the clumpuscule disk has a thickness equal to the H I layer if the velocity dispersion of the dark disk is 1.1 (1.3) times larger than  $\sigma_{\text{H I}}$ , for  $\Sigma_{\text{DM,Kep}} (\Sigma_{\text{DM,Flt}})$ . With these dispersions the “Keplerian dark disk” is close to being stable against radial instabilities ( $Q = 0.8 - 1.2$  beyond the stellar disk) while the “Flat dark disk” is unstable ( $Q = 0.8 - 0.3$ ) in the 5 - 13 kpc range. Here I include a correction factor  $(1 + \frac{2\pi}{R}(\text{FWHM}_{\text{gas,z}}/2.3) \approx 1.3)$  due to the thickness of the disk (e.g., Pfenniger *et al.* 1994). The arguments above suggest that the dark matter may be in the form of a flaring, stable, self-gravitating dark disk with a velocity dispersion slightly larger than the measured dispersion in the H I. It would be very interesting to obtain flaring curves and dispersion measurements for other galaxies to further test the disk-like dark matter hypothesis.

If an anisotropic velocity dispersion ellipsoid of the H I rather than a flattened DM halo is the correct explanation for the data presented in this paper, then this anisotropy would have implications for the physical state of the ISM beyond the optical disk. An anisotropy of the velocity dispersion ellipsoid can oc-

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<sup>9</sup>Note that the dark halo surface densities were calculated in three different ways: Sackett inverts an extrapolated rotation curve, Bosma uses Nordsieck’s (1973) approximation to invert the rotation curve, while Carignan & Puche calculate the DM surface density from a  $z$ -integration through their best fit isothermal DM-halo model.

<sup>10</sup>The flaring of the dark disk is calculated self-consistently using the method described in Paper I (see also Sec. 2.).

cur if, for example, the collision time between “particles” is large.

In accordance with van der Kruit's (1981) finding that the dark halo of NGC 891 cannot be as flattened as the stellar distribution, the axial ratio of NGC 4244's halo (5:1) is smaller than the axial ratio of stellar disk (by a factor of  $\sim 2$  or  $\sim 1.4$  depending on whether the vertically distribution is exponential or  $\text{sech}^2$ , respectively). It seems that the “conspiracy of shapes” (Sackett *et al.* 1994), i.e. the similarity of the shapes of the dark and light distributions, breaks down for the flattest systems. Again, more determinations of the shape of dark halos will make it possible to gauge the frequency of highly flattened halos like the one of NGC 4244.

## 5. CONCLUSIONS

As in Paper I, the potential of the global mass distribution, the self-gravity of the gas included, is used to calculate flaring curves for a series of models with various stellar mass-to-light ratios and dark halo flattenings. Comparing the observed flaring curve with the various model curves I find that the flaring measurements do not constrain the stellar  $M/\mathcal{L}$  beyond the limits imposed by the rotation curve. The shape of the halo on the other hand is well determined. The dark matter halo of NGC 4244 is significantly flattened,  $q_\rho = 0.2^{+0.3}_{-0.1}$  (E9 - E5), albeit less so than the stellar disk. The value for  $q_\rho$  reported here is somewhat smaller than found earlier (Olling & van Gorkom 1995), mainly as a result of a better treatment of the gaseous self-gravity and the use of the measured gaseous velocity dispersion rather than an assumed value. The flaring & velocity dispersion measurements beyond 3 optical scale lengths are consistent with the dark matter being co-spatial with the H I disk, a configuration which is stable against radial instabilities. Uncertainty in the determined inclination is the largest source of random errors. The assumption that the vertical velocity dispersion equals the measured tangential dispersion is the most important potential systematic error: the measured flaring curve is consistent with a round halo if the anisotropy of the gaseous velocity dispersion ellipsoid increases with radius. In that case the vertical dispersion of

the gas has to be 50 - 70% of the measured tangential velocity dispersion at the last measured point.

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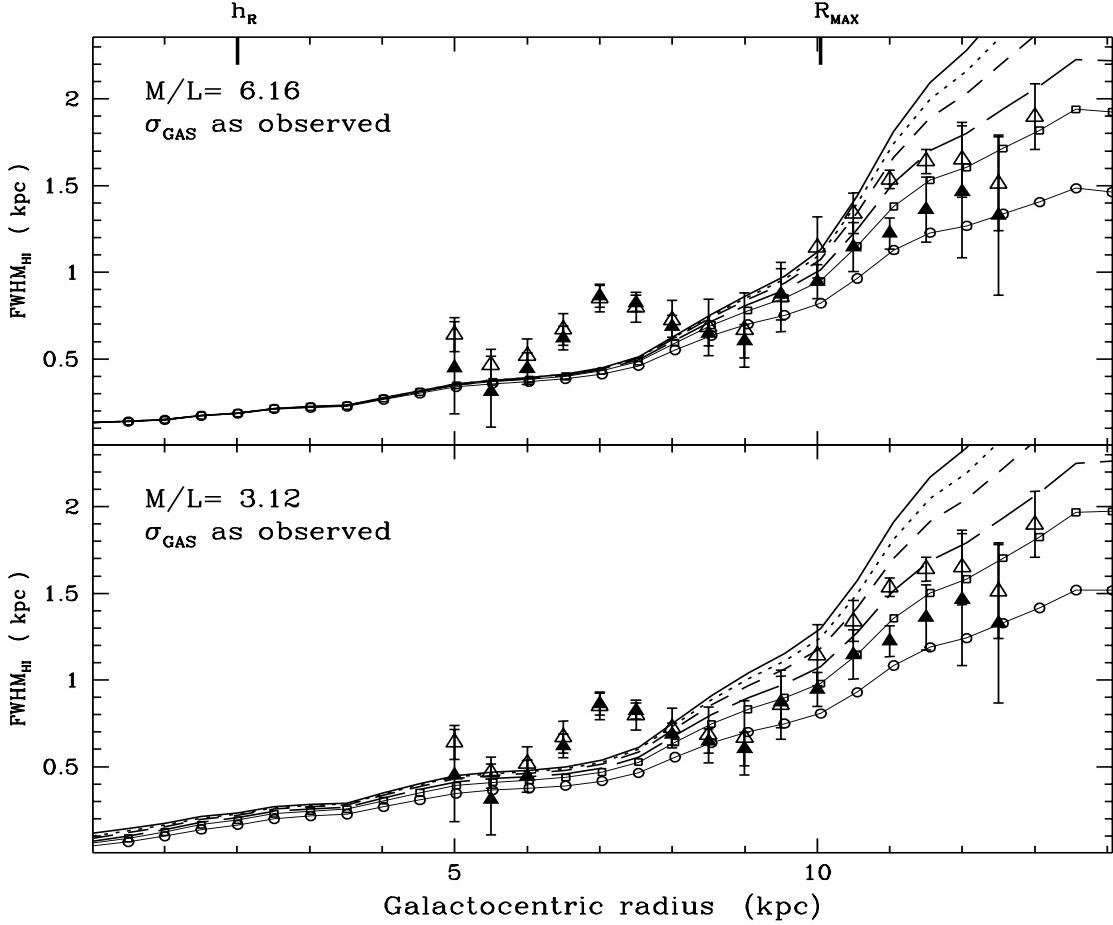


Fig. 1.— The measured widths (open and filled triangles with error bars, for the fixed and varying inclination case described in Paper II, respectively) together with the model widths corresponding to different halo flattenings  $q_\rho$  are plotted:  $q_\rho=1.0, 0.7, 0.5, 0.3, 0.2, \text{ and } 0.1$  from top to bottom. The maximum-disk (top panel) and the minimum-disk (bottom panel) disk-halo decompositions are presented (the structural parameters are given in the caption of Fig. 15 of Paper II). Both the truncation of the stellar disk (at  $R_{\text{MAX}}$ ) and the gaseous self-gravity have been included in the model calculation. The measured gaseous velocity dispersions (Paper II) were used in the model calculations. The observed flaring curve is the average of the north-eastern and south-western sides. Deemed unreliable in the inner 5 kpc (Paper II), the measurements from this region are not plotted. All plotted points are essentially independent measurements. Beyond the optical disk, the thickness of the gas layer is strongly influenced by the shape of the dark halo. Using both inclination cases I find:  $q_\rho=0.2^{+0.3}_{-0.1}$ . The mass-to-light ratio of the stellar disk is not constrained by these flaring measurements.

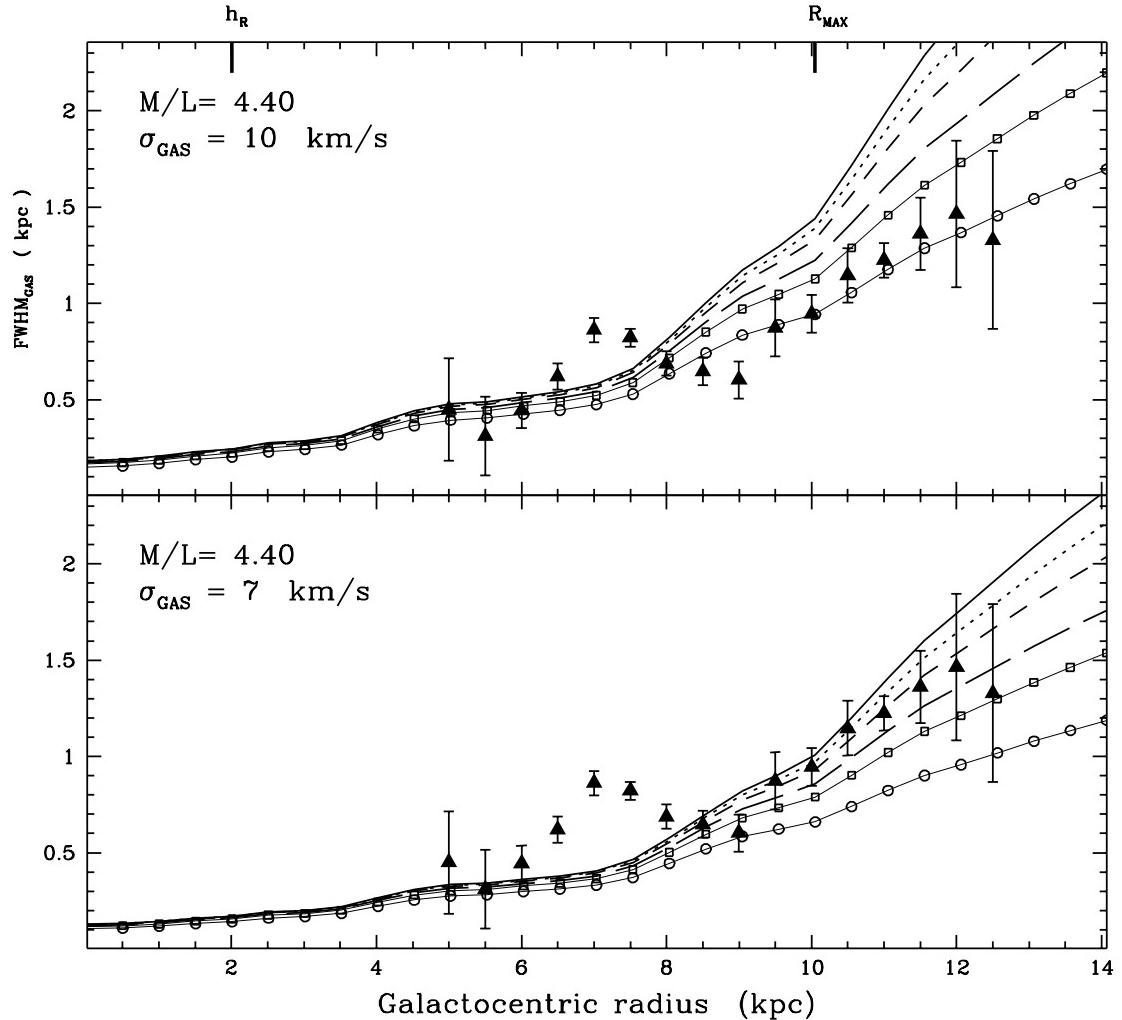


Fig. 2.— The inferred halo flattening depends strongly upon the value of the velocity dispersion. The curves in the top panel (the same labeling of the curves applies as in Fig. 1) are calculated for the “upper limit” of the velocity dispersion as found in the literature. Similarly, the lower panel is calculated for the “lower limit”.

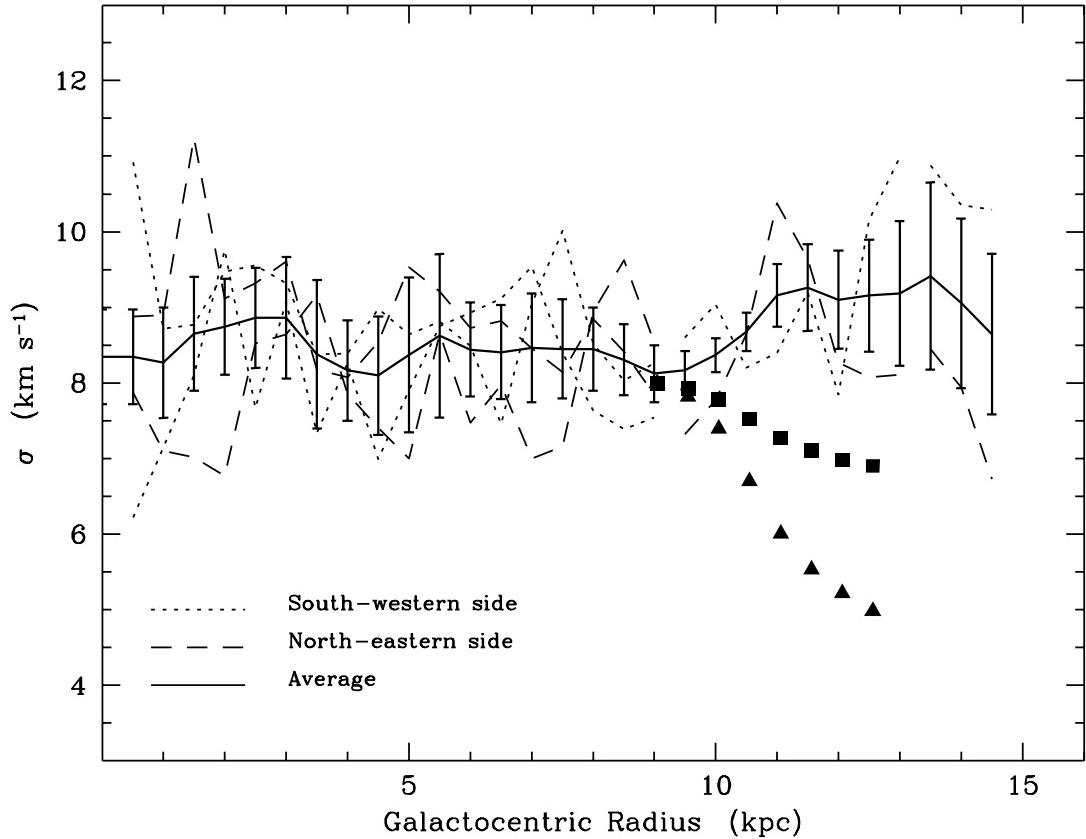


Fig. 3.— The radial variation of the average of the velocity dispersion (full line with the error bars) as determined in Paper II is presented. The dotted lines are determined from the south-western side, the dashed lines from the north-eastern side. For all galactocentric radii, the full line is the average of all curves shown. The filled triangles are the velocity dispersions required to make the model widths for a round halo model consistent with the flaring measurements (see Fig. 1: i.e.,  $\sigma_{zz}$  would decrease by almost a factor two over 3 kpc). To indicate the importance of simultaneously determining the thickness and inclination, I included the required  $\sigma_{zz}$  in case an inclination had to be assumed in determining the thickness of the gas layer (filled squares).